Automata and Grammars
Seminar 10

1. What type of language is generated by grammar G? Is grammar G context free? Find an equivalent context free grammar.

   $G = (\{A, B, C\}, \{a, b, c\}, S, P)$ where P contains the following rules:
   $S \to aSBC|aBC$
   $B \to BBC$
   $C \to CC$
   $CB \to BC$
   $aB \to ab$
   $bB \to bb$
   $bC \to bc$
   $cC \to cc$

2. Consider a context free grammar G. Decide whether string $abcbb$ is generated by G. Use the CYK algorithm for this decision problem.

   $G = (\{A, B, C\}, \{a, b, c\}, S, P)$ where P contains the following rules:
   $S \to CA|CB$
   $C \to ABC|BC$
   $B \to CBA|CB|BA|BB$
   $C \to CC|CB$
   $A \to a$
   $B \to b$
   $C \to c$

3. Suggest conditions which can be used to determine whether a given context free language is infinite or not. Inspire yourself by the pumping lemma for context free languages.

4. Let $b \in \mathbb{N}$. Construct a grammar that generates language $L = \{u\#v\#w | u, v, w \in \{0,1, \dots, b-1\}^* \wedge u_b \circ v_b = w_b\}$ where u_b denotes interpretation of string u as a number with the basis b (that is for example $0101_b = 5$) and $\circ$ some binary arithmetic relation.
   a) $\circ$ is $+$
   b) $\circ$ is –
   c) $\circ$ is $*$
   d) $\circ$ is / (integer division)
   e) $\circ$ is % (modulo)

5. Let $b \in \mathbb{N}$. Construct grammars that generate following languages:
   a) $L = \{w | w \in \{0,1, \dots, b-1\}^* \wedge w_b$ is a composite number $\}$
   a) $L = \{w | w \in \{0,1, \dots, b-1\}^* \wedge w_b$ is a prime number$\}$